



ATAR course examination, 2018

Question/Answer booklet

MATHEMATICS APPLICATIONS

Section Two: Calculator-assumed

Place one of your candidate identification labels in this box.
Ensure the label is straight and within the lines of this box.

Student number: In figures

| | | | | | | | | |
|--|--|--|--|--|--|--|--|--|
| | | | | | | | | |
|--|--|--|--|--|--|--|--|--|

In words

Time allowed for this section

Reading time before commencing work:

ten minutes

Working time:

one hundred minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.



Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|------------------------------------|-------------------------------|------------------------------------|------------------------|-----------------|---------------------------|
| Section One: Calculator-free | 7 | 7 | 50 | 55 | 35 |
| Section Two: Calculator-assumed | 10 | 10 | 100 | 96 | 65 |
| Total | | | | | 100 |

Instructions to candidates

- The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2018*. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answers to the specific question asked and to follow any instructions that are specified to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed**65% (96 Marks)**

This section has **ten** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

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Question 8

(7 marks)

Anthony and Bryan each invest \$4500 in accounts earning compound interest for a period of four years.

(a) Anthony places his money in an account earning interest at the rate of 3.24% per annum, compounded quarterly.

(i) Complete the table below, showing the value of Anthony’s investment at the end of the second and third quarters. (2 marks)

| | | | | | |
|---|---------|---|---|-----|---------|
| Number of quarters money is invested | 1 | 2 | 3 | ... | 16 |
| Value of investment (\$) | 4536.45 | | | ... | 5120.00 |

(ii) State the recursive rule for Anthony’s investment, which gives the values shown in the table above. (2 marks)

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- (b) Bryan places his money in an account earning interest daily. After four years, the value of both Anthony's and Bryan's investments is the same.

Explain how the change to the compounding period has affected the annual rate of interest required for the value of Bryan's investment to be the same as that of Anthony.

Include calculations to support your answer.

(3 marks)

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Question 9

(8 marks)

Deborah is purchasing mealworms for her pet lizard, Lizzy, to eat.

Deborah starts by buying 50 mealworms. She then buys an additional 15 at the start of each subsequent week. She feeds 12 mealworms to Lizzy each week, and each week a certain percentage of the mealworms dies.

Deborah has found that the approximate number of mealworms at the start of the n^{th} week can be modelled by M_n , where $M_{n+1} = 0.9(M_n - 12) + 15$, $M_1 = 50$.

- (a) What percentage of the mealworms dies each week? (1 mark)
- (b) Determine the approximate number of mealworms Deborah has at the start of the fifth week. (1 mark)
- (c) Deborah claims that she will never run out of mealworms using this model. Justify her claim. (2 marks)

After 10 weeks, hot weather results in a larger percentage of the mealworms dying, so Deborah alters the model to:

$$N_{n+1} = 0.8(N_n - 12) + 15, N_1 = c$$

(d) (i) Determine the value of c . (1 mark)

(ii) Determine the approximate number of mealworms Deborah has at the start of the thirtieth week. (1 mark)

Deborah's vet recommends feeding Lizzy 10 mealworms a week. She would also like to maintain a constant number of 30 mealworms at the start of each week, so she changes the above model to:

$$P_{n+1} = 0.8(P_n - 10) + k$$

(e) Determine the value of k , the number of mealworms she must buy each week, to ensure this occurs. (2 marks)

Question 10

(8 marks)

Sales of new motor vehicles in Australia, December 2016

| | | Passenger vehicles | Sports utility vehicles | Other vehicles | Total |
|-------------------------------------|------------------------|--------------------|-------------------------|----------------|--------|
| Western Australia | Number of vehicles | 3093 | 3087 | 1846 | 8026 |
| | Percentage of vehicles | 38.5% | 38.5% | 23.0% | 100.0% |
| Victoria | Number of vehicles | 11985 | 9575 | A | 27065 |
| | Percentage of vehicles | B | 35.4% | 20.3% | 100.0% |
| Australian Capital Territory | Number of vehicles | 875 | 585 | 212 | 1672 |
| | Percentage of vehicles | 52.3% | 35.0% | 12.7% | 100.0% |
| New South Wales | Number of vehicles | 15005 | 11788 | 6839 | 33632 |
| | Percentage of vehicles | 44.6% | 35.0% | 20.3% | 100.0% |
| Queensland | Number of vehicles | 7668 | 6822 | 4762 | 19252 |
| | Percentage of vehicles | 39.8% | 35.4% | 24.7% | 100.0% |
| South Australia | Number of vehicles | 2720 | 2197 | 1547 | 6464 |
| | Percentage of vehicles | 42.1% | 34.0% | 23.9% | 100.0% |
| Tasmania | Number of vehicles | 795 | 648 | 475 | 1918 |
| | Percentage of vehicles | 41.4% | 33.8% | 24.8% | 100.0% |
| Northern Territory | Number of vehicles | 244 | 234 | 256 | 734 |
| | Percentage of vehicles | 33.2% | 31.9% | 34.9% | 100.0% |
| Australia (Total) | Number of vehicles | 42385 | 34936 | 21442 | 98763 |
| | Percentage of vehicles | 42.9% | 35.4% | 21.7% | 100.0% |

Note: As percentages have been rounded correctly to one decimal place, totals of percentages may not add to exactly 100%.

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Use the information in the table to answer the following questions:

- (a) Determine the values of **A** and **B** for the Victorian data. (2 marks)
- (b) Compare the percentage of the total new vehicle sales in Western Australia with those in South Australia. (3 marks)
- (c) Describe the association between the number of sales of new passenger vehicles and new sports utility vehicles in Australia. (1 mark)
- (d) Compare and comment on the percentage sales of vehicles in the Northern Territory with those in other States/Territories. (2 marks)

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Question 11

(8 marks)

Charles turns 60 years of age today. He has been saving for his retirement when he reaches 65 and currently has \$465 000 to put toward this. His plan is to have an amount of \$675 000 by the time he retires, and to set up a pension fund from which he can be paid an annuity each year.

- (a) Charles invests his money in an account earning interest at the rate of 3.35% per annum, with interest calculated and added to his account at the end of each month. He also deposits an additional amount of money at the end of each month. Determine the monthly deposit required by Charles if he is to reach his goal by his 65th birthday. (3 marks)

Assuming Charles successfully manages to save the \$675 000 by his 65th birthday, he will set up his pension fund. It is anticipated that the fund will earn interest at the rate of 3.25% per annum, compounded monthly, and he will withdraw an annuity of \$65 000 each year on his birthday.

- (b) (i) Determine the number of years that he will be able to receive this annuity. (3 marks)
- (ii) Charles is hopeful that it will be possible for him to continue receiving an annuity until his 85th birthday. He decides to find an alternative fund offering a different interest rate, while continuing to withdraw \$65 000 each year. What annual interest rate would he need to receive to make his money last until his 85th birthday? (2 marks)

Question 12

(6 marks)

A service centre manager recorded the number of customers over time periods, t , and produced the following spreadsheet to compare different moving averages.

| t | Number of customers | 3-point moving average | 4-point centred moving average | 5-point moving average | 6-point centred moving average |
|-----|---------------------|------------------------|--------------------------------|------------------------|--------------------------------|
| 1 | 840 | | | | |
| 2 | 927 | 901 | | | |
| 3 | 936 | 919 | 902.625 | 892.8 | |
| 4 | 894 | 899 | 893.625 | 890.4 | C |
| 5 | 867 | 863 | 879 | 888.6 | 895 |
| 6 | 828 | 871 | 880.875 | 886.8 | 890.25 |
| 7 | 918 | 891 | 886.5 | 883.8 | 882 |
| 8 | 927 | 908 | 891 | B | 874.5 |
| 9 | 879 | 886 | 881.625 | 879 | 877 |
| 10 | 852 | 850 | 866.625 | 876.6 | 883.75 |
| 11 | 819 | 859 | 869.25 | 875.4 | 878.75 |
| 12 | 906 | 882 | 876 | 872.4 | 869.75 |
| 13 | 921 | 897 | 879.375 | 868.8 | 863 |
| 14 | 864 | 873 | 870 | 868.2 | |
| 15 | 834 | 838 | | | |
| 16 | A | | | | |

- (a) What is the purpose of calculating moving averages for time series data? (1 mark)
- (b) Determine the values **A**, **B** and **C** in the above table. (3 marks)
- (c) From those in the table above, which is the most appropriate moving average for the manager of the service centre to consider? Justify your choice. (2 marks)

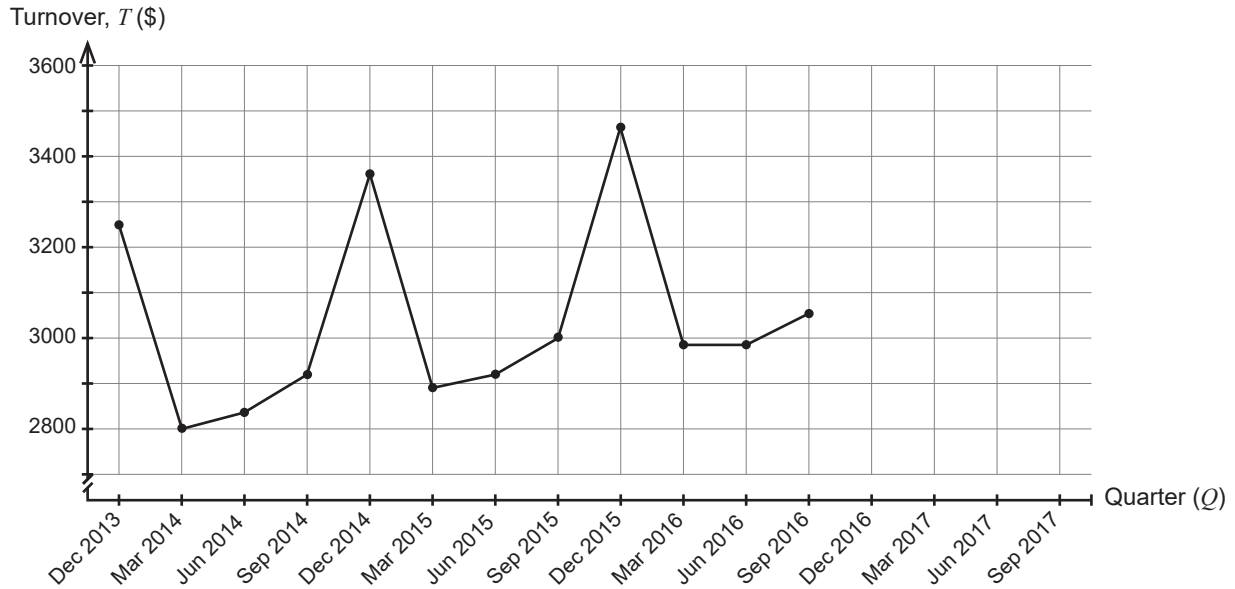
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Question 13

(13 marks)

The graph below shows the quarterly retail turnover per capita (\$) in Australia, i.e. the average amount spent per person at retail outlets during each quarter.

Quarterly retail turnover per capita, Australia, 2013–2017



The data for the next four quarters are shown in the following table.

| Quarter | December 2016 | March 2017 | June 2017 | September 2017 |
|---|---------------|------------|-----------|----------------|
| Quarterly retail turnover per capita (\$) | 3521.40 | 2980.10 | 3045.00 | 3075.30 |

- (a) Complete the time series plot by including this additional information. (2 marks)

- (b) The equation of the least-squares line for the above data is $T = 9.6143Q + 2986.50$, where $Q = 1$ for December 2013, $Q = 2$ for March 2014, etc.
 - (i) Fit this line to the graph. (2 marks)

 - (ii) Describe the trend and seasonality of this data. (2 marks)

- (c) The 4-point centred moving average for March 2017 is \$3152.78 (correct to two decimal places). Determine the **actual** retail turnover per capita for September 2016. (2 marks)

- (d) The seasonal indices (correct to two decimal places) are shown in the table below.

| Quarter | Seasonal index |
|-----------|----------------|
| December | 110.76% |
| March | 95.00% |
| June | |
| September | 98.20% |

- (i) Complete the table by determining the seasonal index for June. (1 mark)
- (ii) Use the seasonal index to determine the deseasonalised retail turnover per capita for December 2016. (2 marks)
- (iii) The deseasonalised retail turnover per capita for March 2016 is \$3142.42. Determine the **actual** retail turnover per capita for this quarter. (2 marks)

Question 14

(12 marks)

Marco is a plumber. Three years ago, he purchased a vehicle costing \$48 000 for his business. He paid a deposit of \$5000 and acquired a personal loan for the remainder from a financial institution, at a reducible interest rate of 22.5% per annum, compounded monthly. He agreed to make repayments of \$1000 at the end of each month.

- (a) (i) Use a recurrence relation to determine the amount Marco currently owes on the loan. (3 marks)
- (ii) Determine how much longer it will take him to completely pay off the loan. (2 marks)
- (b) After three years, Marco finds that his vehicle is only worth \$27 150. Determine the average rate of depreciation of his vehicle, expressed as a percentage. (2 marks)

- (c) When Marco originally took out a personal loan for the purchase of his vehicle, he was given two options by the financial institution. These were:
- increasing his monthly repayment by \$200, or
 - taking an option of reducing the interest rate to 18.5% and maintaining repayments of \$1000 per month.

In terms of time taken to pay off the loan and total paid for his vehicle, which should he have chosen and why? (5 marks)

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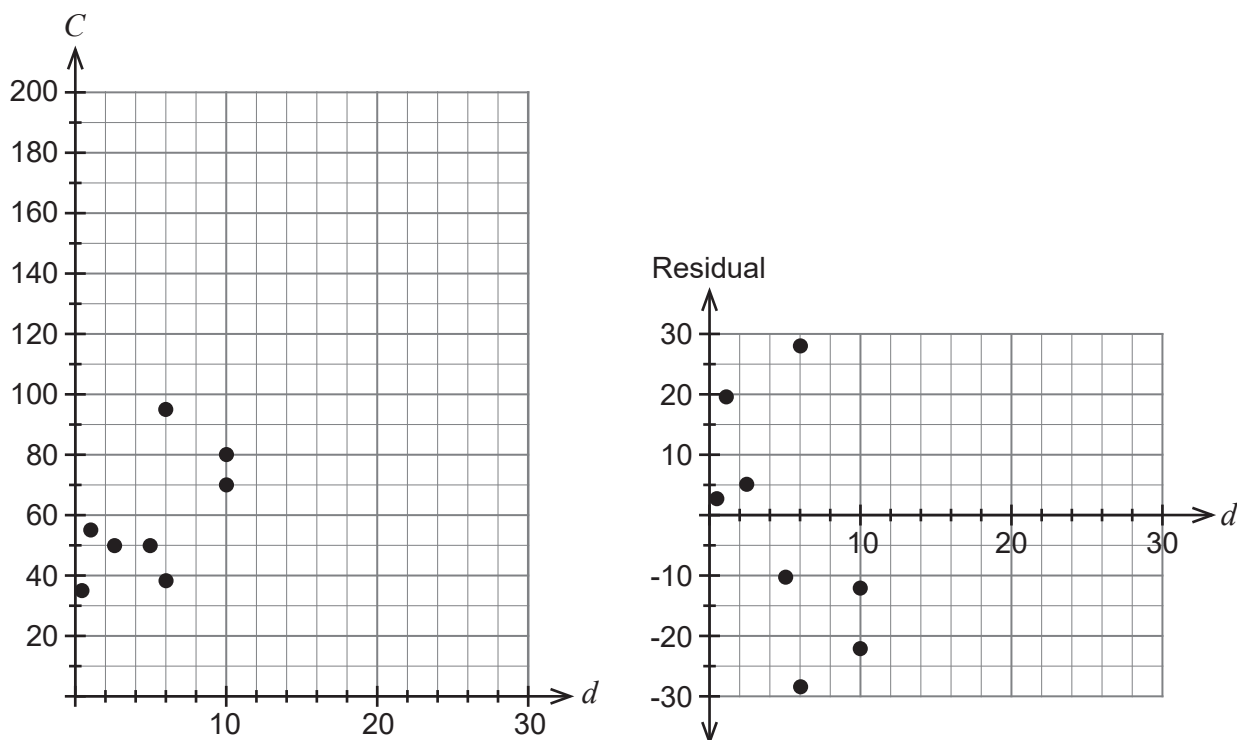
Question 15

(12 marks)

Ali is researching mobile phone carriers and has found several plans with monthly contracts. The table below shows the data allowance, GB (d) and the monthly cost \$ (C), of ten plans that he is considering.

| | | | | | | | | | | |
|---|----|-----|-----|-----|----|----|----|----|-----|----|
| Data allowance GB (d) | 10 | 2.5 | 0.5 | 15 | 5 | 1 | 6 | 6 | 25 | 10 |
| Monthly cost \$ (C) | 70 | 50 | 35 | 135 | 50 | 55 | 95 | 38 | 195 | 80 |

The graphs below show a scatterplot and a residual plot for the information in the table, with two points missing on both graphs.



(a) Plot the two missing points on the scatterplot. (2 marks)

(b) (i) Determine the equation of the least-squares line for the information in the table and state the correlation coefficient. (2 marks)

- (ii) Describe the linear association between Data allowance and Monthly cost. (2 marks)
- (iii) Approximately how much does the cost change for every additional GB of data allowance? (1 mark)
- (iv) What percentage of the variation in monthly cost can be explained by the variation in the data allowance? (1 mark)
- (c) (i) Calculate the two missing residuals and include them on the residual plot. (2 marks)
- (ii) What feature of the residual plot indicates that a linear model would be appropriate for the data? (1 mark)
- (d) Predict the monthly cost of a plan with a data allowance of 20 GB. (1 mark)

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Question 16

(11 marks)

Natalia inherits a sum of money from her grandfather. She wishes to place it in a high-interest savings account.

She is considering the following two options:

- Account A: interest rate 4.40% per annum, compounded monthly
- Account B: interest rate 4.30% per annum, compounded daily.

- (a) The effective annual interest rate for Account A is 4.49% (correct to two decimal places). Determine the effective annual interest rate for Account B. (1 mark)

Natalia’s bank offers her another account, C, with an interest rate of 4.50% per annum.

- (b) Under what circumstances will this interest rate and the effective annual interest rate be the same? (1 mark)

- (c) Which account (A, B or C) should Natalia choose to maximise her savings? Explain your reasoning. (2 marks)

Natalia’s sister, Elena, has inherited \$25 000 from her grandfather. She decides to invest this money in a high-interest savings account, with interest compounded monthly. Elena also chooses to deposit an additional \$250 into this account at the end of each month.

The table below shows Elena’s account balance over the first three months.

| Month | Account balance at start of month | Interest earned | Deposit | Account balance at end of month |
|-------|-----------------------------------|-----------------|----------|---------------------------------|
| 1 | \$25 000.00 | \$125.00 | \$250.00 | \$25 375.00 |
| 2 | \$25 375.00 | \$126.88 | \$250.00 | \$25 751.88 |
| 3 | \$25 751.88 | \$128.76 | \$250.00 | \$26 130.64 |

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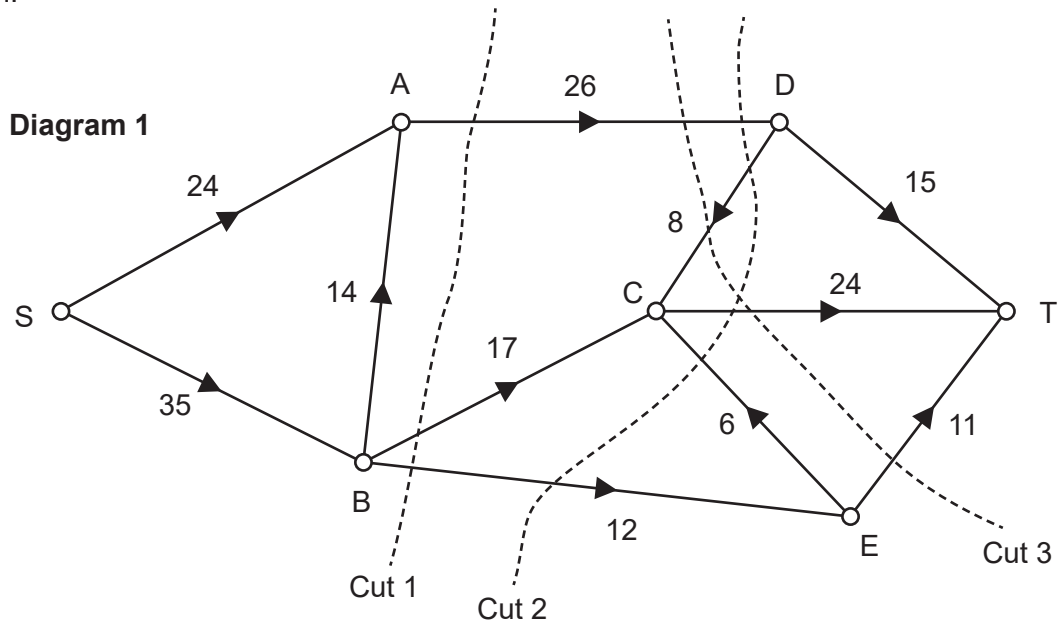
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- (d) Show that the annual interest rate that applies to Elena's account is 6%. (1 mark)
- (e) The amount in Elena's account, A_n at the end of month n , can be expressed as a recursive rule, $A_{n+1} = cA_n + d$, $A_0 = 25\,000$. Determine the values of c and d . (2 marks)
- (f) After two years, Elena wishes to use the money she has saved as a deposit for a house. An amount of \$35 000 will be required. Unfortunately, Elena has realised that by depositing \$250 each month she will **not** reach her savings goal.
- (i) If she only deposits \$250 each month, by how much will she be short of the required deposit? (2 marks)
- (ii) What increase in the monthly deposit is required for Elena to save the \$35 000 in two years? (2 marks)

Question 17

(11 marks)

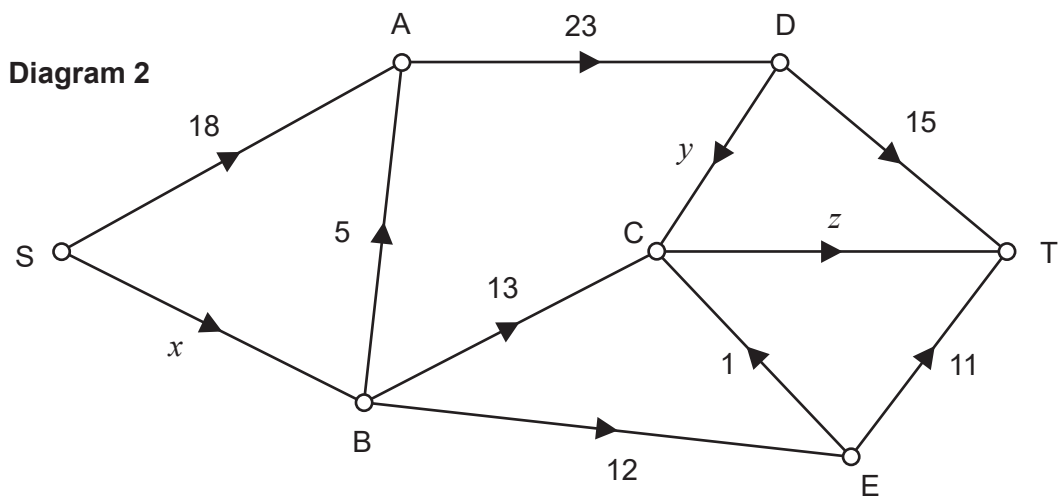
Diagram 1 shows a network of pipes. The number on each edge gives the capacity of that pipe in L/min.



(a) State the capacities of the three cuts in Diagram 1.

(3 marks)

Diagram 2 shows a possible flow for the network of pipes.



See next page

- (b) (i) Explain why the value of x is 30. (1 mark)
- (ii) Calculate the values of y and z . (2 marks)
- (c) State which of the pipes are at full capacity in Diagram 2. (2 marks)
- (d) State the value of the flow for the network in Diagram 2. (1 mark)
- (e) (i) The value of the flow for Diagram 2 can be increased by 2 L/min. List the series of pipes that could be used to achieve this. (1 mark)
- (ii) Show that the increased flow in part (e)(i) is a maximum for this network of pipes. (1 mark)

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End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

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ACKNOWLEDGEMENTS

Question 10 Table data source: Australian Bureau of Statistics. (2017). *Table 2: New motor vehicles sales by state, type, all series*. Retrieved April, 2018, from <http://www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/9314.0December%202016?>
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Question 13 Graph data source: Australian Bureau of Statistics. (2018). *Table 19: Quarterly retail turnover per capita, Australia, all series*. Retrieved April, 2018, from <http://www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/8501.0Nov%202017?>
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